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Multi-objective optimization under uncertain objectives

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Outline

1 – Introduction: MOP in uncertain environment

→State of the art

2 – Proposed formulation

→To take into account the uncertainties of objective functions

3 – Case study

→Applications & Results

4 – Discussion

→Limits & Future work

Questions ?

1 - Introduction

EMO : Optimization in uncertain Environments [Jin & Branke 2005],…

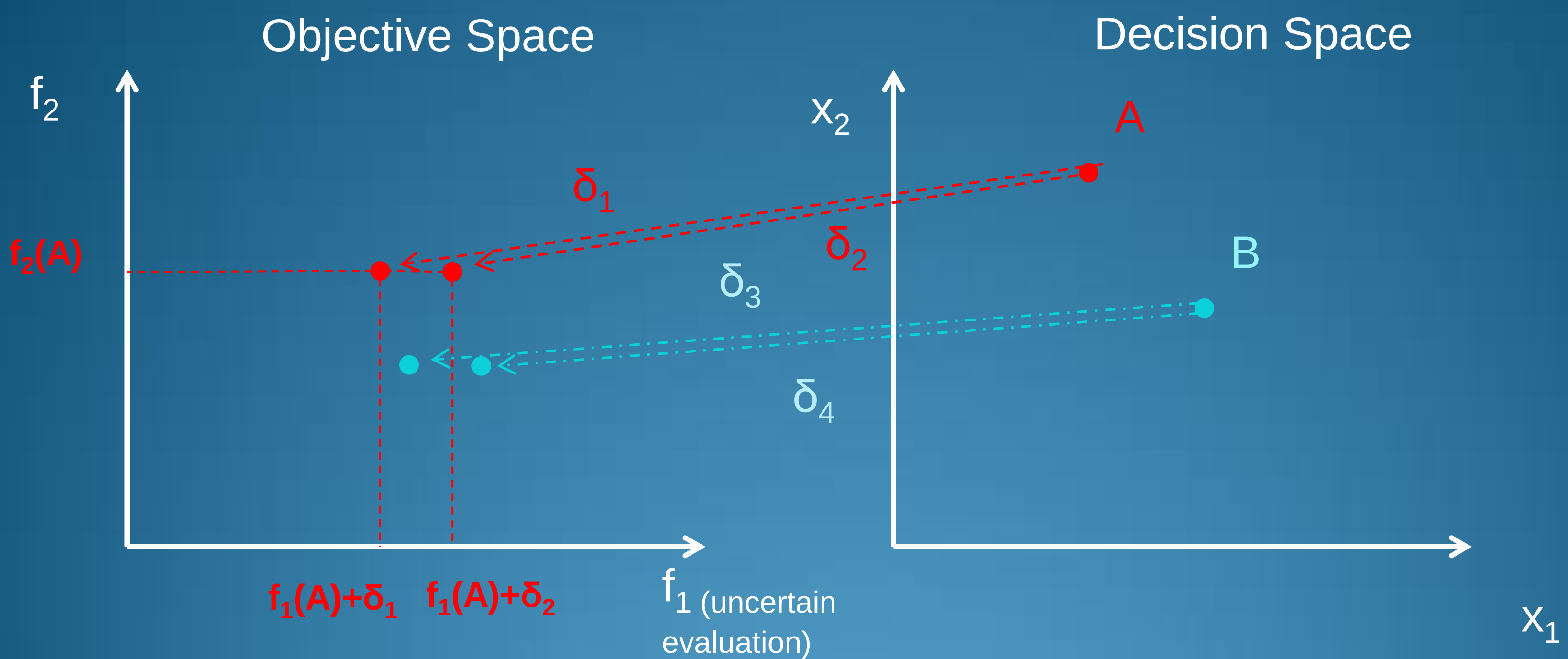
Uncertainties on evaluation of objective functions (Noise)	Uncertainties on decision variable (Robustness)	Fitness Approximation (Meta-model)	Dynamic environments
[Tan & Goh 2008],…	[Deb & Gupta 2006],…	[Jin & Sendhoff 2002],…	[Tfailli 2007],…
$f(x) \rightarrow f(x)+\delta$	$f(x) \rightarrow f(x+\delta)$	$f(x) \rightarrow f(x)+E(x)$	$f(x) \rightarrow f_t(x)$

Example: $\delta \sim N(0,\sigma^2)$

$E(x)$: deterministic error

Example with noise: $f_1(x) \rightarrow f_1(x)+\delta$ (minimization problem)

δ is different for each evaluation of each individual during the optimization



Who dominates who ?

→ Uncertain performances

→ Convergence problem

1 - Introduction

EMO : Optimization in uncertain Environments

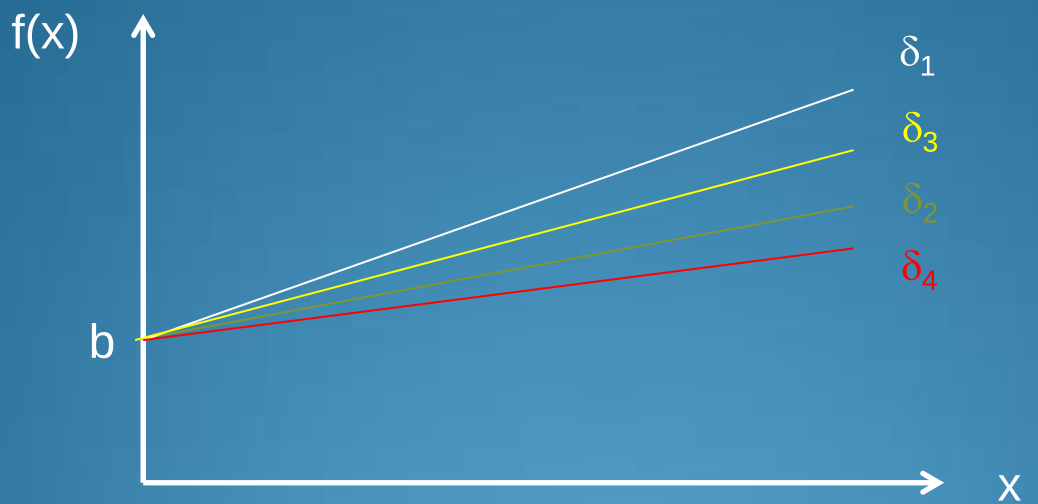
- Uncertainties on evaluation of objective functions (Noise)
  - Explicit averaging
    [Babbar et al. 2003],…
    Many evaluations (average, worst case,…)
  - Implicit averaging
    [Goldberg et al. 1992],…
    Increasing population (mistakes averaged)
  - Modification of the algorithm
    [Branke & Schmidt 2003],…
    Ranking, selection,…
- Uncertainties on decision variable (Robustness)
- Fitness Approximation (Meta-model)
- Dynamic environments

The goal is to converge to the Pareto Front that would have been obtained without noise

New case: Uncertain objective function

→ Objective function defined with parameters that are uncertain

Example: $f_\delta(x) = \delta x + b$ with δ an uncertain parameter

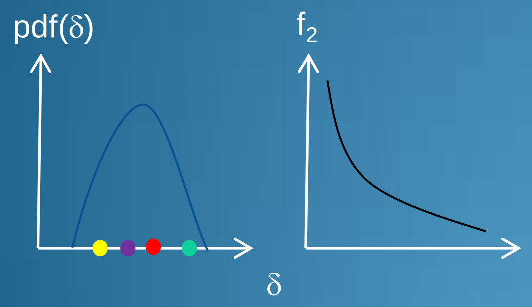


Other examples: tolerance about material properties, cost, …

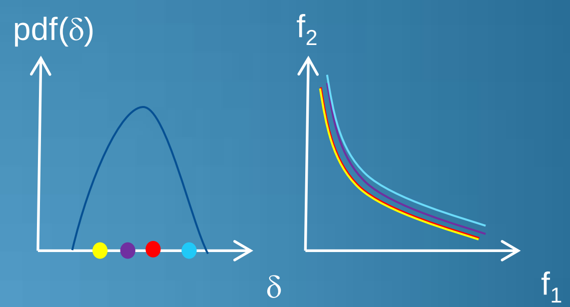
→ Different from objective function affected by noise

Uncertain objective function: new kind of uncertainty

Noise	Uncertain objective function
$f(x) \rightarrow f(x)+\delta$	$f(x) \rightarrow f_\delta(x)$
One optimization is performed for eliminating the effect of the uncertainties (i.e. find Front de Pareto without noise)	Various optimizations are performed for extracting knowledge from uncertainties affecting objective functions
δ is different for each evaluation of each individual during the optimization	f_δ is the same for all evaluations for all individuals during an optimization f_δ is different for each optimization



Number of generation * population * number evaluations different δ



Number optimizations different δ

Noise	Uncertain objective function
Noisy evaluation	Deterministic evaluation
A single Pareto front is obtained at the end of process	Various Pareto fronts are obtained at the end of process

1 - Introduction

EMO : Optimization in uncertain Environments

Uncertainties on evaluation of objective functions (Noise)

Uncertain objective functions

Uncertainties on decision variable (Robustness)

Fitness Approximation (Meta-model)

Dynamic environments

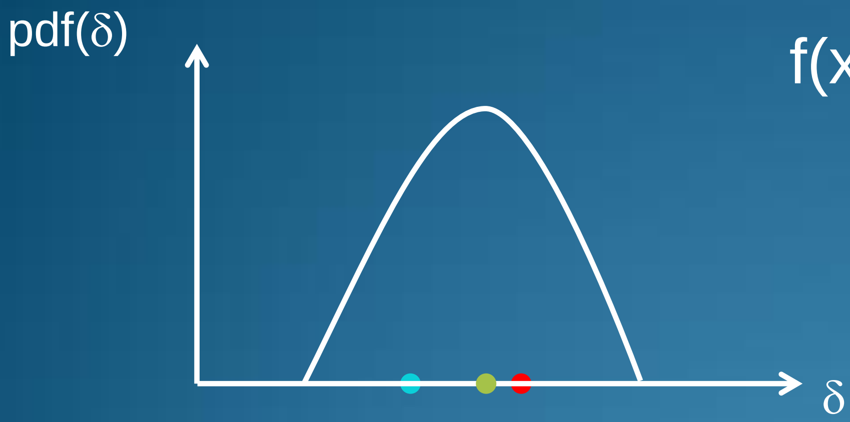
Extract informations and knowledge from uncertainties

Try to eliminate the effects of noise

In what follows we consider a **probabilistic framework**

Assumption: uncertain parameters will be defined as random variable according to their probability density functions (**pdf**)

Behavior when uncertain parameters are random variables

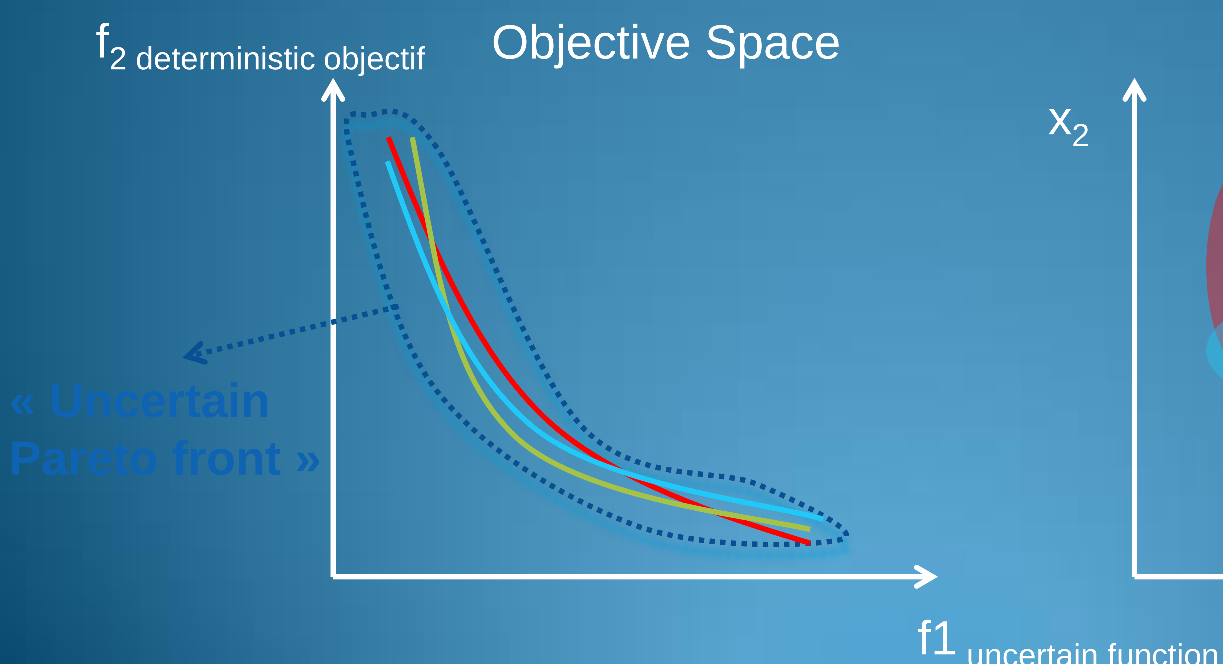


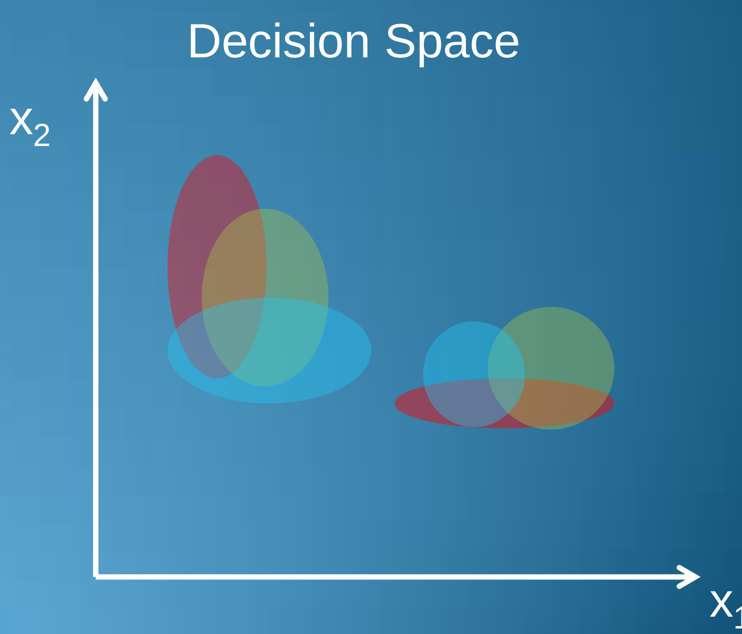
$f(x) \rightarrow f_{\delta}(x)$

$f(x) \rightarrow f_{\delta_1}(x)$

$f(x) \rightarrow f_{\delta_2}(x)$

$f(x) \rightarrow f_{\delta_3}(x)$





How to extract information and knowledge from the previous process ?

$\delta \rightarrow f_{\delta}$

Objective Space

"Tradeoff probability function"

$$P_t(T) = \int_{\delta / T \in F} pdf(\delta) d\delta$$

Probability that the tradeoff T to be one of the best tradeoffs (i.e. T belongs to Pareto front)

Decision Space

"Solution probability function"

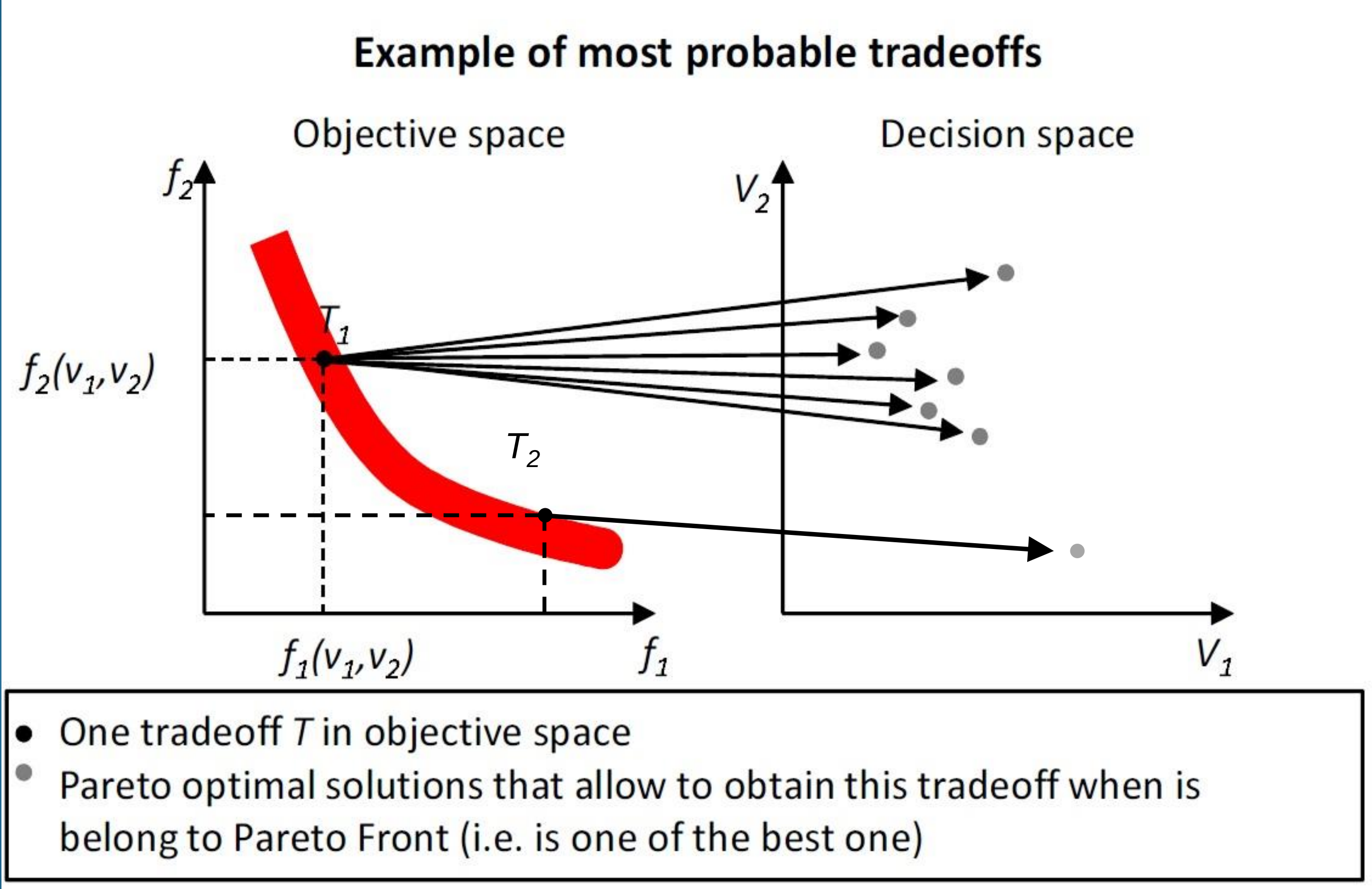
$$P_d(S) = \int_{\delta / S \in SF} pdf(\delta) d\delta$$

Probability that the solution S to belongs to the set of Pareto optimal solutions

Approaches to obtain information through the ”tradeoff probability function”

→ Probable tradeoffs

$P_t(T_1)>P_t(T_2)$ → T_1 is more probable than T_2

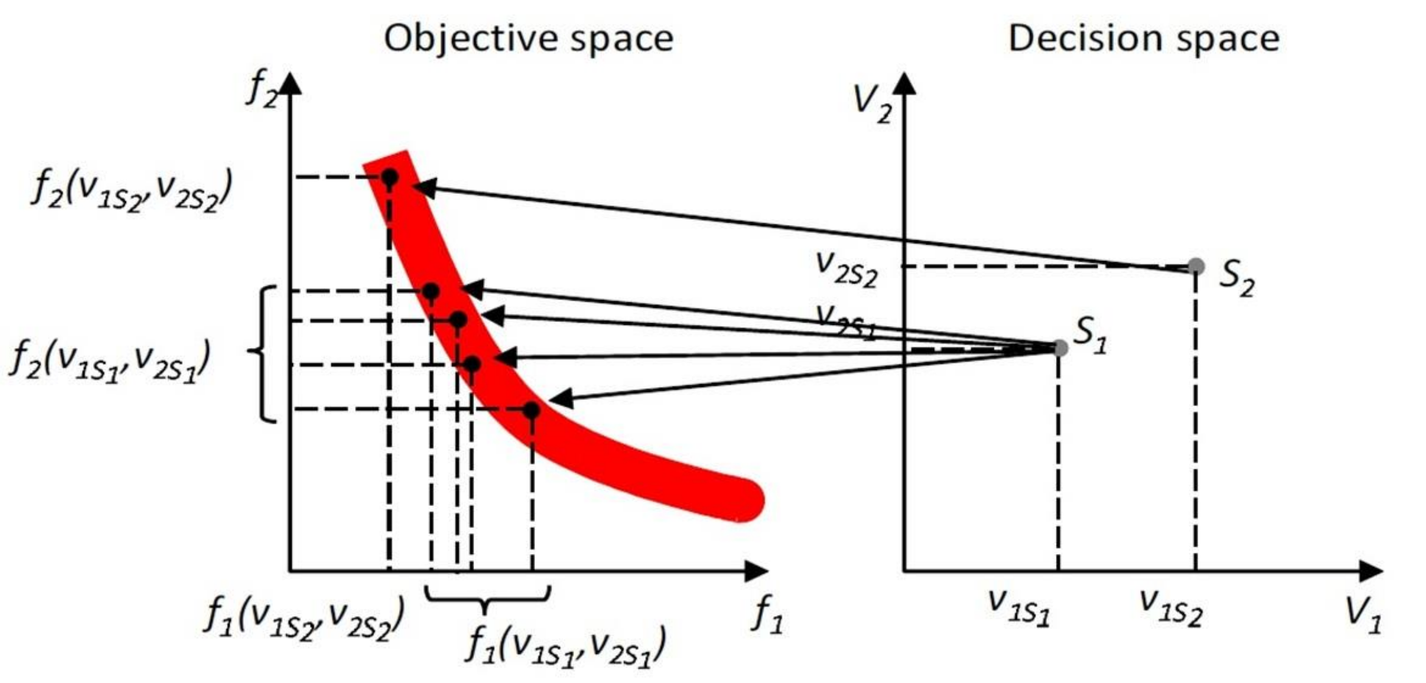


Approaches to obtain information through the ”solution probability function”

→Reliable solutions

$P_d(S_1)>P_d(S_2)$ → S_1 is more reliable than S_2

Example of reliable solutions



- Pareto optimal solutions S_1 and S_2 in decision space
- Best tradeoffs in objective space that can be obtained when S_1 or S_2 is Pareto optimal

Algorithm

Draw a f_δ according the pdf of δ

uncertain objective function → an objective function (possible and determined)

Resolving the EMO optimization

F_i : Pareto front

SF_i : Pareto optimal solution

Loop

« Uncertain Pareto Front » = $\lim\limits_{n \to \infty} \sum_{i=1}^{n} F_i$ i.e. Meeting of all possible Pareto fronts

→ Approximated by Monte Carlo draws

Approximated by *n* Monte Carlo draws & Discretization of Spaces

Objective Space

"Tradeoff probability function"

$$P_t(px) = \frac{\sum_i^n \delta_{F_i,px}}{n} \quad \text{with} \quad \delta_{F_i,px} = \begin{cases} 1 & \text{if } F_i \in \text{px} \\ 0 & \text{if otherwise} \end{cases}$$

Decision Space

"Solution probability function"

$$P_d(sx) = \frac{\sum_i^n \delta_{SF_i,sx}}{n} \quad \text{with} \quad \delta_{SF_i,sx} = \begin{cases} 1 & \text{if } SF_i \in \text{sx} \\ 0 & \text{if otherwise} \end{cases}$$

Likewise

P(T/S)

And

$P(S \cap T)=P_d(S)*P(T/S)$

equal

Likewise

P(S/T)

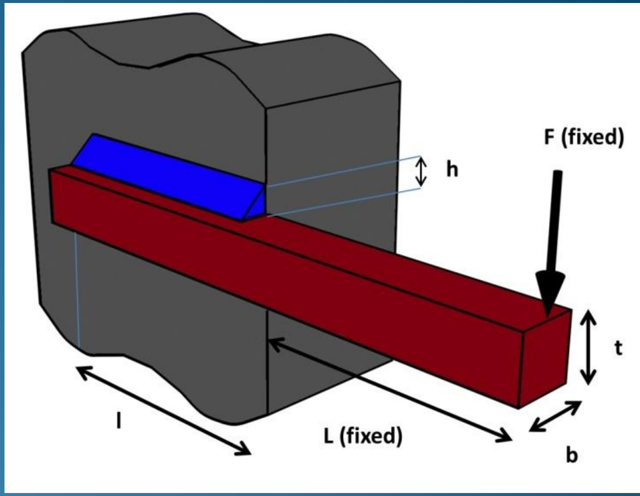
And

$P(T \cap S)=P_t(T)*P(S/T)$

Notation : p(A/B) is the probability of event A given event B equivalent of p(A|B)

Case study: welded beam design problem

- Four decision variables (inch) :
  - h : the thickness of the weld
  - l : the length of the weld
  - t : the width of the beam
  - b : the thickness of the beam

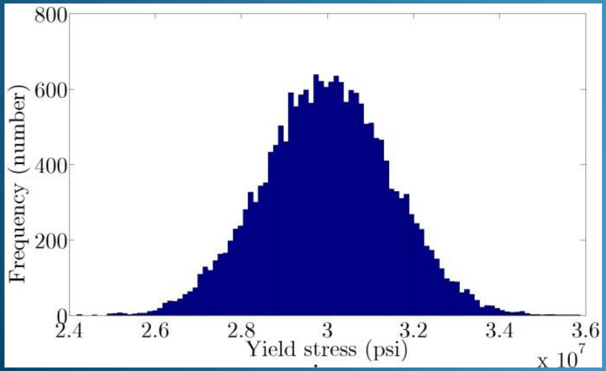


- Two objective functions : the cost ($) and the end deflection (inch)

- Four constraints : $g_1 \rightarrow g_4$

- Uncertain Young's modulus

$E \sim \text{Normal}\,(30.10^6, 9.10^{12})$

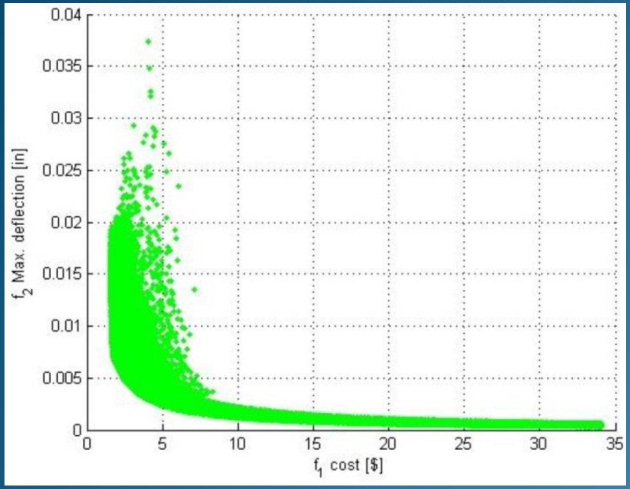


$$\begin{cases} f_1(x) = 1.104h^2 l + 0.048tb(14+l) \\ f_2(x) = \dfrac{2.1952}{t^3 b} = \dfrac{4FL^3}{\boxed{E}t^3 b} \end{cases}$$

f_2 uncertain function

$$\begin{cases} g_1(x) = 13600 - \tau(x) \geq 0 \\ g_2(x) = 30000 - \sigma(x) \geq 0 \\ g_3(x) = P_c(x) - 6000 \geq 0 \\ g_4(x) = b - h \geq 0 \\ \textit{with} \\ \tau(x) = \sqrt{(\tau')^2 + (\tau'')^2 + \dfrac{l\tau'\tau''}{\sqrt{0.25(l^2+(h+t)^2}}} \\ \textit{where} \\ \tau' = \dfrac{6000}{\sqrt{2}hl} \\ \tau'' = \dfrac{6000(14+0.5l)\sqrt{0.25(l^2+(h+t)^2)}}{2(0.707hl(\frac{l^2}{12}+0.25(h+t)^2))} \\ \sigma(x) = \dfrac{504000}{t^2 b} \\ P_c(x) = 64746.022(1-0.0282346t)tb^3 \end{cases}$$

20.000 draws of Monte Carlo



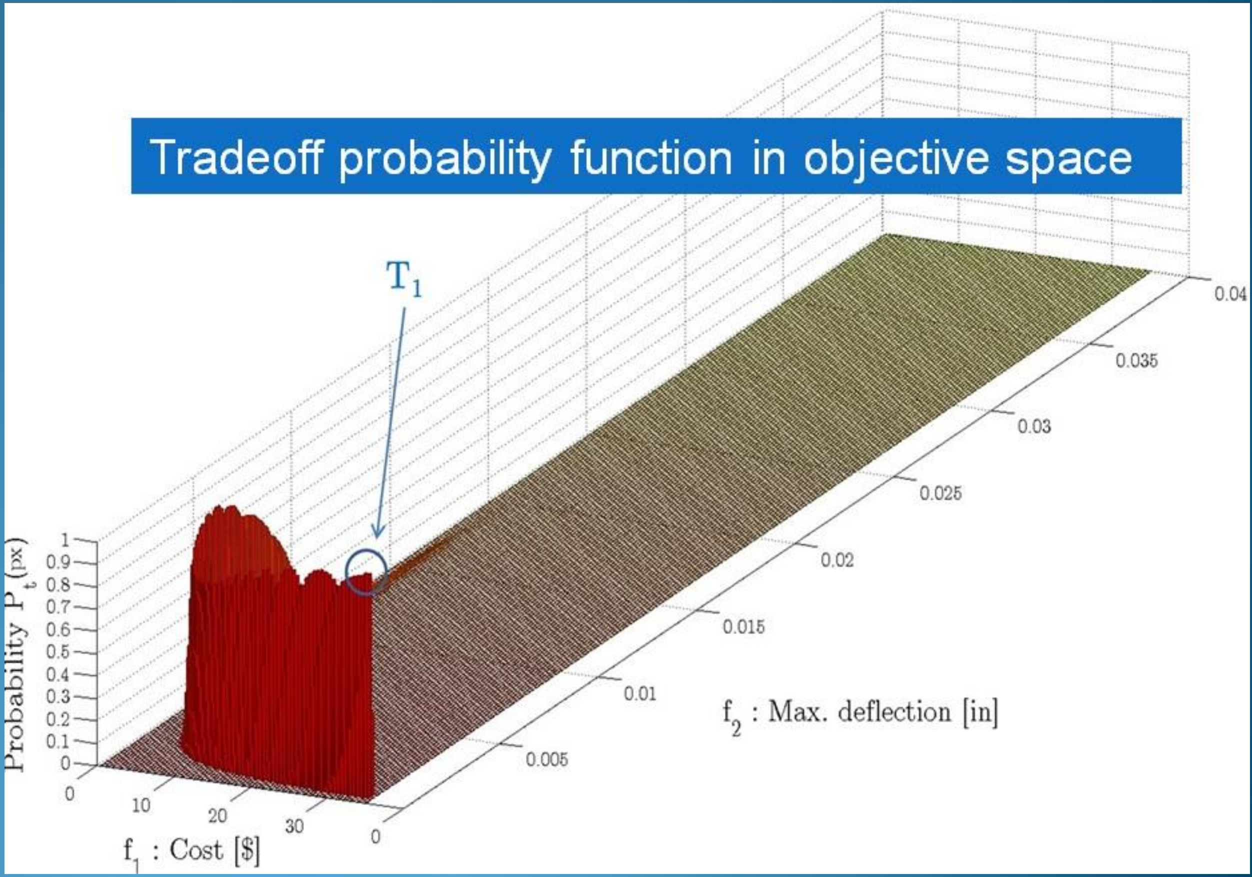
Objective space		Decision space (inch)
$f_1(\$)$	$f_2(inch)$	h l t b
0.5	0.0001	0.1

Size of discrete elements

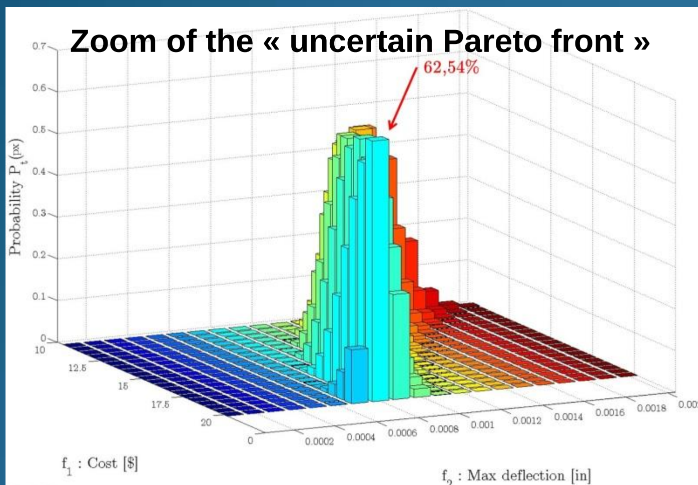
Example with T_1

$T_1(69,5)$: $f_1 \varepsilon [34,35]$
$f_2 \varepsilon [0.0004,0.0005]$

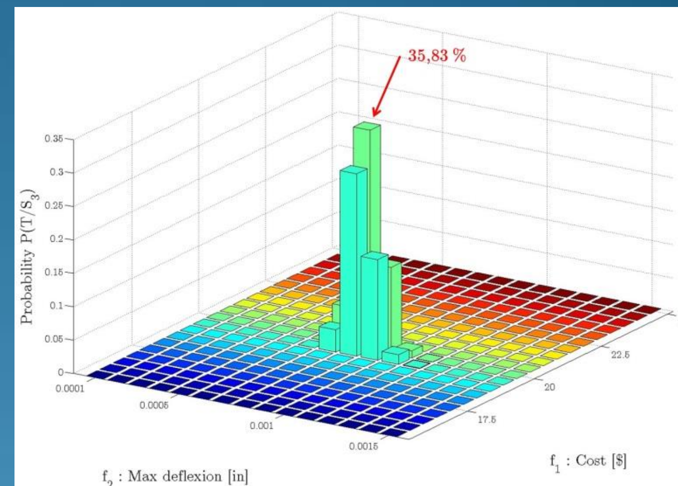
$p_t(T_1) = 0.85635$



Moreover we know the other likely tradeoffs with a particular solution and therefore its other possible performances, so the range of possible performances of the solution



Most probable tradeoff with f_1<20$



Tradeoffs with a particular solution

The « Uncertain Pareto front »

→ Provides all information on the influence of uncertain objective functions

→ Supplies the decision maker : Most probable tradeoff
Most reliable solution
Probabilities of tradeoff associated with a solution

Generic method : any evolutionary algorithm can be used

Applications to other fields (psychophysical functions)

Future work:

Decrease the computing time ?

→ Use of other numerical schemes like Metropolis-Hastings

Ensure the convergence of the optimization algorithm ?

→ Estimation of convergence error

Study relationship with uncertainties related to decision variables (robustness)?

→Use of effective uncertain function

Thank you for your attention …

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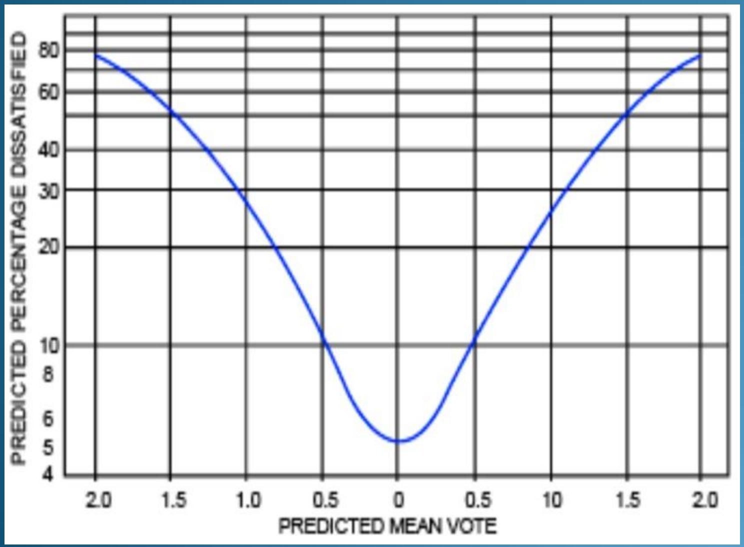
Examples of applications

Sustainable development

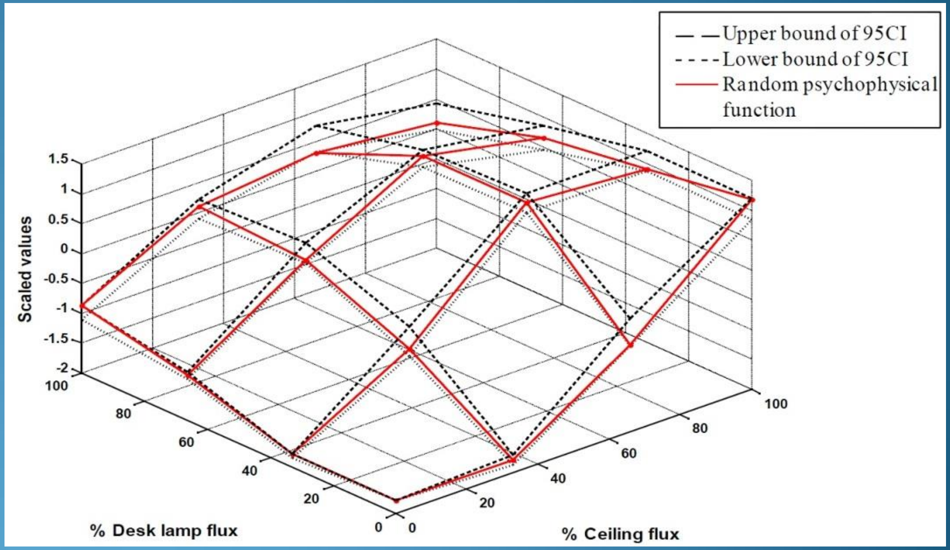
→Take into account the users in building construction

→visual, thermal, acoustic psychophysical functions

→ Uncertain psychophysical functions



Thermal comfort



Visual comfort